

Physics 129a
Set # 5

Solutions to Problem

1. Perkins 3.8

First combine 1st 2, then add 3rd

a) $\pi^+ \pi^- \pi^0$

$$\pi^+ \pi^- = |I^{(1)}=1 I_3=1\rangle |I^{(2)}=1 I_3=-1\rangle =$$

$$\sqrt{1/6} |I=2 I_3=0\rangle + \sqrt{1/2} |I=1 I_3=0\rangle + \sqrt{1/3} |I=0 I_3=0\rangle$$

now combine w/ $I=1, I_3=0$. This will give us

$$\frac{1}{\sqrt{6}} (\sqrt{3/5} |130\rangle + \sqrt{2/5} |110\rangle) +$$

$$\frac{1}{\sqrt{2}} (\sqrt{2/3} |120\rangle - \sqrt{1/3} |100\rangle) +$$

$$\frac{1}{\sqrt{3}} (|110\rangle)$$

$$= \sqrt{1/10} |130\rangle + \sqrt{1/3} |120\rangle + (\sqrt{1/3} - \sqrt{4/15}) |110\rangle - \sqrt{1/6} |100\rangle$$

$$\boxed{\text{so } I = 3, 2, 1, 0}$$

b) $\pi^0 \pi^0 \pi^0$

$$\pi^0 \pi^0 = |I^{(1)}=1 I_3=0\rangle |I^{(2)}=1 I_3=0\rangle$$

$$= \sqrt{2/3} |120\rangle - \sqrt{1/3} |100\rangle$$

now combine w/ $|I=1 I_3=0\rangle$

$$= \sqrt{2/3} (\sqrt{3/5} |130\rangle - \sqrt{2/5} |110\rangle$$

$$- \sqrt{1/3} |110\rangle)$$

$$= \sqrt{2/5} |130\rangle - \sqrt{1/3} |110\rangle - \sqrt{4/15} |110\rangle$$

$$\boxed{\text{so } I = 3, 1}$$

2. Perkins 3.10

$$\text{a) } \rho^0 \rightarrow \pi^+ \pi^-$$

$$J=1 \quad J=0 \quad J=0 \Rightarrow \ell=1$$

Parity $(-1)^{\ell} (-1)^2$ for final state = -1
so parity is conserved

$$\text{Isospin } |I^{(1)}=1, I_3=1\rangle, |I^{(2)}=1, I_3=-1\rangle = \\ \sqrt{1/6} |I=2, I_3=0\rangle - \sqrt{1/2} |I=1, I_3=0\rangle + \sqrt{1/3} |I=0, I_3=0\rangle$$

so $I=1$ state is ok

allowed by strong interactions

$$\text{b) } \rho \rightarrow \pi^0 \pi^0$$

Spin + parity argument same as above

$$\text{but } |I^{(1)}=1, I_3=0\rangle, |I^{(2)}=1, I_3=0\rangle$$

$$= \sqrt{2/3} |I=2, I_3=0\rangle \neq \sqrt{1/3} |I=0, I_3=0\rangle$$

\Rightarrow no $I=1$ state

not allowed by strong interactions

Also if $\ell=1$ bose statistics don't work
since $w_f \rightarrow -1 \times w_f$ under interchange \Rightarrow not EM

$$\text{c) } \eta^0 \pi^0$$

$$\eta \text{ has } I=0 \quad J^{PC}=0^{-+}$$

ρ has $C = -$ π^0 has $C = +$

so initial $C = -$ final $C = +$

not allowed by strong or EM

$$\text{a) } \rho \rightarrow \pi^0 \kappa$$

Not allowed by Strong int since κ in final state

$C_{\text{int}} = -1$ $C_{\text{final}} = -1$ so C_{ISOK}
can happen via EM.

$$3. \quad \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\text{a) } \lambda_1 + \lambda_2 = 2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_1 - \lambda_2 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 + \lambda_5 = 2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 - \lambda_5 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_6 + \lambda_7 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_6 - \lambda_7 = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore (\lambda_1 + \lambda_2) u = 2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$d = 2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2u$$

$$s = 2 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$(\lambda_1 - \lambda_2) u = 2 \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2d$$

$$d = 2 \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$s = 2 \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$(\lambda_4 + \lambda_5) u = 2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$d = 2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$s = 2 \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2u$$

$$(\lambda_4 - \lambda_5) u = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 2s$$

$$d = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0$$

$$c \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

$$(\lambda_6 + \lambda\lambda_1) u = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$d = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 0,$$

$$s = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 2d$$

$$(\lambda_6 - \lambda\lambda_1) u = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 0$$

$$d = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = 2s$$

$$s = 2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

so $(\lambda_1 \pm \lambda\lambda_2)/2$ is $\left\{ \begin{matrix} \text{raising} \\ \text{lowering} \end{matrix} \right\}$ op between
u+d

$(\lambda_4 \pm \lambda\lambda_5)/2$ is $\left\{ \begin{matrix} \text{raising} \\ \text{lowering} \end{matrix} \right\}$ op between
u+s

and $(\lambda_6 \pm \lambda\lambda_7)/2$ is $\left\{ \begin{matrix} \text{raising} \\ \text{lowering} \end{matrix} \right\}$ op between
d+s

b) $\Delta^+ = \frac{1}{\sqrt{3}} (uu d + udu + duu)$

$$\Delta^0 = \frac{(\lambda_1 - \lambda\lambda_2)}{2} \Delta^+$$

$$= \frac{1}{\sqrt{3}} \underbrace{(udd + duu + udd + duu + dud + ddu)}_2$$

$$= \frac{1}{\sqrt{3}} (udd + dud + ddu)$$

c To change Δ^+ to state w/ 2 u's and 1 n
apply $(\lambda_6 - \lambda\lambda_7)/2$

$$\frac{(\lambda_6 - \lambda\lambda_7)}{2} \Delta^+ = \frac{1}{\sqrt{3}} (uus + usu + sun)$$

4. a) $m_{K^0} = 498 \text{ MeV}$ $m_{K^{*0}} = 892 \text{ MeV}$

Since $m_1 + m_2$ the same for $K + K^*$ and
by def Σ_{binding} is same, the only
difference is the $A \frac{s_1 s_2}{m_1 m_2}$ term.

$$(s_1 + s_2)^2 = s_1^2 + s_2^2 + 2 s_1 \cdot s_2 \Rightarrow$$

$$s_1 \cdot s_2 = \frac{1}{2} ((s_1 + s_2)^2 - s_1^2 - s_2^2)$$

$$\text{for } s=1 = \frac{1}{2} ((1)(2) - \frac{1}{2}(\frac{3}{2}) - \frac{1}{2}(\frac{3}{2})) = \frac{1}{2}(2 - \frac{3}{2}) \\ = \frac{1}{4}$$

$$\text{for } s=0 = \frac{1}{2} (0 - \frac{1}{2}(\frac{3}{2}) - \frac{1}{2}(\frac{3}{2})) = \frac{1}{2}(-\frac{3}{2}) = -\frac{3}{4}$$

$$\text{so } \frac{A}{m_1 m_2} \underbrace{\left(\frac{1}{4} - \left(-\frac{3}{4} \right) \right)}_{=1} = 892 - 498 \text{ MeV} \\ = 394 \text{ MeV}$$

using $m_d = 363 \text{ MeV}$ $m_s = 538 \text{ MeV}$

$$A = (394 \text{ MeV})(538 \text{ MeV})(363 \text{ MeV}) = 7.69 \times 10^7 \text{ MeV}$$

b ω has spin 1 + quark content $\frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d})$
 η has spin 0 + quark content
 $\frac{1}{\sqrt{6}} (d\bar{d} + u\bar{u} - 2s\bar{s})$

Because quark content is different, we cannot approximate as well as in part a.
But, let's do our best:

compare $\eta + K^0$:

$$\begin{aligned}\Delta m(\eta - K^0) &= \frac{2}{6} (m_d - m_s) + \frac{4}{6} (m_s - m_d) \\ &\quad + \left(-\frac{3}{4}\right) \frac{A}{m_d m_s} \left[\frac{2m_s}{6m_d} + \frac{4m_d}{6m_s} \right] \\ &= 58 + [-1] \\ &= 24 = 82 \text{ meV}\end{aligned}$$

$$\begin{aligned}\Delta m(W - K^+) &= -m_s + m_d + \frac{1}{4} \frac{A}{m_d m_s} \left[\frac{m_s}{m_u} - 1 \right] \\ &= -175 + 47 \\ &= -127 \text{ meV}\end{aligned}$$

$$\begin{aligned}\therefore \Delta m(W - \eta) &= \Delta m(K^+ - K^-) - 127 - 82 \\ &= 394 - 127 - 82 \\ &= 185 \text{ meV}\end{aligned}$$

correct answer:

$$m_W = 182 \text{ meV}$$

$$m_K = 549 \text{ meV}$$

$$\Delta m = 233 \text{ meV}$$

$$\begin{aligned}
 5.a) \Delta m(D^+ - D^-) &= \frac{A}{m_u m_s} \left(\frac{m_s}{m_c} \right) \\
 &= (394 \text{ MeV}) \left(\frac{538}{1500} \right) \\
 &= 141 \text{ MeV}
 \end{aligned}$$

correct ans (PPG): $2007 - 1865 = 142 \text{ MeV}$

$$b) \Delta m(D_s^+ - D^0) = ?$$

$$\begin{aligned}
 \Delta m(D_s^+ - D_s^-) + \Delta m(D_s^- - D^0) &= \\
 \frac{A}{m_u m_s} \left(\frac{m_u}{m_c} \right) + m_s - m_d & \\
 = (394 \text{ MeV}) \left(\frac{363 \text{ MeV}}{1500 \text{ MeV}} \right) + 538 - 363 & \\
 = 270 \text{ MeV} & \quad "95 \text{ MeV}
 \end{aligned}$$

True ans (PDG) = $2112 - 1865 = 247 \text{ MeV}$

not bad.